The Role of Porosity in Filtration VIII: Cake Nonuniformity in Compression–Permeability Cells

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Compression-permeability cells are utilized for measurement of porosity and permeability of compressible cakes. Shortcomings of the cells have tended to be minimized in the literature. Data are presented which show that the stress distribution in typical C-P cells is highly nonuniform because of wall friction. For compressible materials, the porosity and permeability would also be nonuniform. Use of large L/D ratios and minimizing wall friction are essential to proper use of C-P cells.

Local flow resistance has been employed for many years in development of filtration formulas. Ruth (*14*) introduced the compression-permeability (C-P) cell as a device for direct measurement of local porosities and resistances. Since then the C-P cell has served as an important tool in conducting filtration research. Grace (*3*) presented the first correlation of extensive sets of C-P cell data with results obtained from constant pressure filtration. During the last decade in a series of papers on filtration behavior of clay materials, Shirato and co-workers (*11, 15* to *17*) utilized C-P test data to predict the performance of both constant pressure and constant rate filtration. Ingmanson (*5*) and Kottwitz and Boylan (*8*) also reported that average filtration resistances could be predicted from results of C-P cell experiments.

Shortcomings of the C-P cell tend to be minimized in published materials. Drawing on previous work in soil mechanics, Grace (*3*) concluded that wall friction in the C-P cell would not be significant if the ratio L/D of cake thickness to cell diameter did not exceed 0.6. Working with a 2-in. diameter cell, Haynes (*4*) found that 20 to 60% of applied load was consumed as side wall friction in 1 to 3-in. cakes. Lu et al. (*10*) reported substantial increase in specific filtration resistance with time and questioned reliability of the cell as a universal testing device.

Although investigators have recognized the existence of wall friction and the importance of technique, correlations have been based upon the assumption that friction could be neglected. Utilizing simplified assumptions concerning friction between the cake and cell wall, Tiller, Haynes, and Lu (*18*) developed expressions for correction factors for porosity and resistance. They showed experimentally that the fraction of applied load transmitted to the bottom of a cake in a C-P cell was a function of both cell diameter and applied load as well as the ratio L/D of cake thickness to cell diameter. Their results were at variance with those of Rawling (*12*) who attempted to obtain permeabilities and porosities free of wall effect by extrapolating L/D to zero. Rawling's method was questionable because he assumed that L/D was the only parameter.

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Sawamoto (*16*) studied the simplified equations proposed by Haynes (*4*) and concluded that they had limited application. Sawamoto (*16*) and Lu (*9*) found that wall friction did not vary linearly with applied load.

Reliability of the C-P cell depends on how closely local conditions in actual filtration are duplicated in the cell. There are a number of sources of inaccuracy, the most important being side wall friction which causes nonuniform cakes, lag in time required to reach equilibrium, change of cake characteristics with time, and method of cake preparation. This paper will be concerned with an analysis of side wall friction as it affects cake uniformity in the C-P cell.

STRESS DISTRIBUTION AND CAKE NONUNIFORMITY

Knowledge of stress distribution in confined particulate solid beds is a key to the understanding of side wall friction data in C-P cell tests. Wall friction causes nonuniformity of stress distribution which in turn affects porosity and permeability. Investigators have analyzed stress distribution in confined particulate solid systems in two different manners. In the classic treatment (*1, 6, 15, 16*) the cake is treated as a continuous medium. In a newer approach (*2, 7, 13*), the bed is handled as a particulate mass. Neither method gives quantitative results which are adequate for analysis of a cake in a C-P cell.

Measurement of horizontal and vertical pressures in small confined cakes is difficult. For the experimental measurement of stresses in a moving solid system, Delaplaine (*1*) developed a device for measuring the horizontal pressure as well as the vertical pressure at the wall. Tschebotarioff and Welch (*19*) carried out experimental measurements on the lateral pressure distribution along the wall of a compressibility cell. Their purpose was to determine how the lateral pressure could be related to the vertical pressure. Unfortunately, they obtained the relationship only at the top surface of the sample. Sawamoto (*16*) measured the distribution of vertical pressure transmitted to the system of a 3-in. diameter cell and reported that it was uniformly transmitted. His results are at variance with those presented in this paper. Nevertheless until more data are available, the probability exists that pressure is more uniformly transmitted with some solids than with

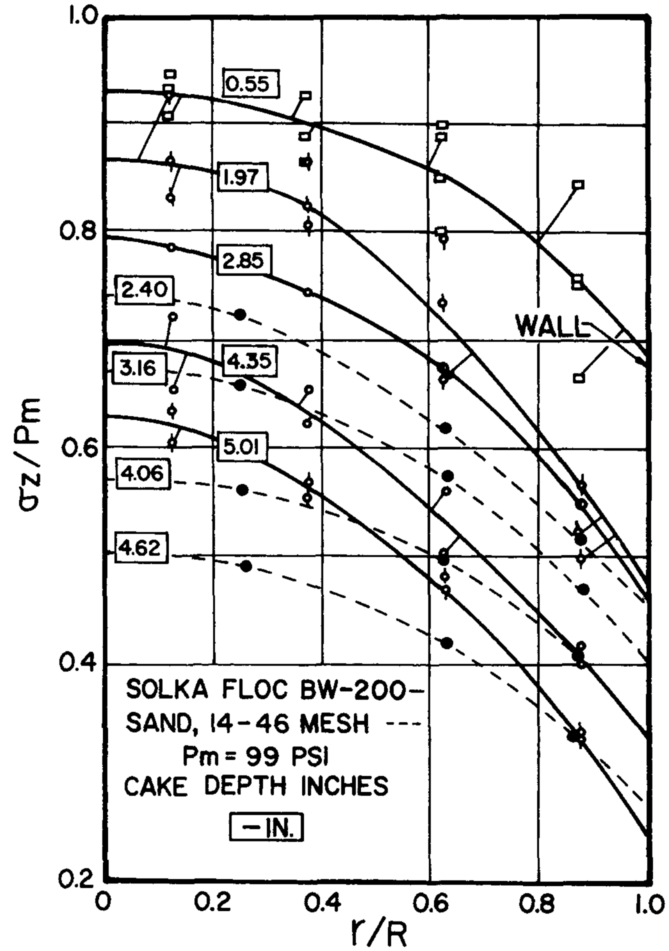

Fig. 1. Vertical pressure distribution on septum. Pressure versus radius.

others. Structural uniformity of cakes is favored by a low coefficient or friction between the solids and the cell wall.

EXPERIMENTAL EQUIPMENT

In order to examine the stress distribution in cakes, a 4-in. diameter C-P cell was designed to give horizontal pressures at various depths and the vertical pressure distribution at the bottom septum.

The horizontal pressure was measured by means of plugs placed in slots and connected to load cells equipped with strain gauges. To measure the radial distribution of the vertical stress at the bottom of the cake, the porous septum was constructed of four concentric rings. Each ring had three brass tubes (with attached strain gauges) for support except the center one which had a single tube.

Materials used in the experiments consisted of a highly compressible alpha cellulose (Solka Floc) and a noncompressible 14-40 mesh sand. To form a bed, a highly concentrated slurry was first prepared. At zero compressive pressure, Solka Floc has a porosity ranging from 0.85 to 0.90. At those porosities, the cake is relatively soupy. A 13.9% by weight slurry corresponding to a porosity slightly in excess of 0.90 was prepared and placed under vacuum to assure that no air bubbles would be formed in the cake. As sand is noncompressible, no serious problems are encountered in preparation of a cake. It is only necessary to make sure that the minimum porosity is reached by loading or vibration.

HORIZONTAL AND VERTICAL PRESSURE DISTRIBUTION

A series of horizontal and vertical pressure distribution measurements were carried out for Solka Floc BW-200 and sand. In Figure 1, the vertical pressure distributions obtained from different cake thicknesses are presented. The applied mechanical pressure was not uniformly transmitted to the septum. The thicker the cake, the more nonuniform was the vertical pressure distribution. As expected, the maximum vertical pressure was measured at the center ring. The horizontal pressure distributions along the wall of the C-P cell are shown in Figure 2. Not all of the curves or all of the points are shown for the Solka Floc. The curves were extrapolated to both the cake surface and the septum in order to get the horizontal wall pressures at the top ($z = 0$) and the bottom ($z = L$). Under the same mechanical pressure of 99.0 lb./sq.in., the ratio σ_r/P_m at the wall when extrapolated to the surface ranged from 0.27 to 0.40 for Solka Floc with most values close to 0.33 to 0.35. Values of 0.43 to 0.47 were obtained for sand.

RATIO OF HORIZONTAL TO VERTICAL PRESSURE AT WALL K_0

The ratio of horizontal to vertical pressure is known as the coefficient K_0 of lateral earth pressure in soil mechanics. It is given by

$$\sigma_r = K_0 \sigma_z \tag{1}$$

Many investigators have assumed K_0 to be constant. However, Jaky (*6*) assumed that K_0 increased with the depth of

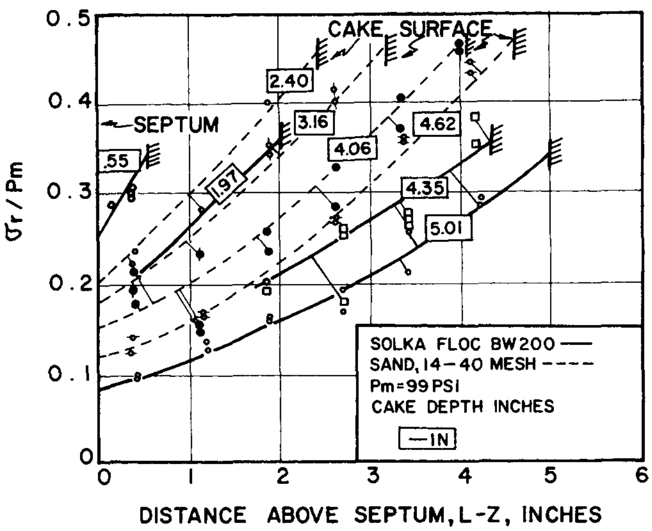

Fig. 2. Horizontal pressure distribution on side wall. Pressure versus height.

TABLE 1.

Solka Floc 99 lb./sq.in.

Thickness, in.	σ_z/P_m at $r = D$	σ_r/P_m at $z = L$	K_0 col. 3/col. 2	K_0 σ_r/P_m at $z = 0$
0.55	0.69	0.27	0.403	0.31
1.97	0.48	0.175	0.365	0.35
2.35	0.46	0.125	0.272	0.33
4.35	0.335	0.115	0.344	0.36
5.01	0.245	0.085	0.346	0.345
Sand 99 lb./sq.in.				
2.40	0.445	0.195	0.447	0.45
3.16	0.40	0.175	0.438	0.45
4.06	0.335	0.150	0.446	0.46
4.62	0.265	0.125	0.471	0.465

bed in a silo holding dry grain.

As indicated in the last section, the values of both vertical and horizontal pressures at the wall can be obtained at both top and bottom surfaces by extrapolating the pressure distribution curves. The ratio of the extrapolated values represents K_0. Values obtained by extrapolation from Figures 1 and 2 are summarized in Table 1.

The values are quite sensitive to slight changes in the slopes of the curves. A more reliable method (*18*) consists of plotting the logarithm of σ_r/P_m at the wall against the depth z as illustrated in Figure 3. The points shown in Figure 3 were obtained from the smooth curves drawn in Figures 1 and 2 and are not data points. Simplified theory (*18*) indicates that the depth-stress relationship follows the equation

$$\sigma_z = \sigma_r/K_0 = e^{-4K_0fz/D} \tag{2}$$

Thus a semilogarithmic plot of σ_r versus z yields an intercept of K_0 at $z = 0$, and the slope gives K_0f. From Figure 3, values of $K_0 = 0.35$ and $f = 0.78$ for Solka Floc and $K_0 = 0.46$ and $f = 0.62$ for sand are obtained. The frictional coefficients would appear to be higher than might be expected.

STRESS DISTRIBUTION IN THE C-P CELL

Utilizing the data presented, it is possible to calculate the vertical and horizontal stresses at the boundaries of the C-P cell. The internal stress distribution can then be roughly estimated. In turn, some idea of the variation of porosity and nonuniformity of cake can be obtained.

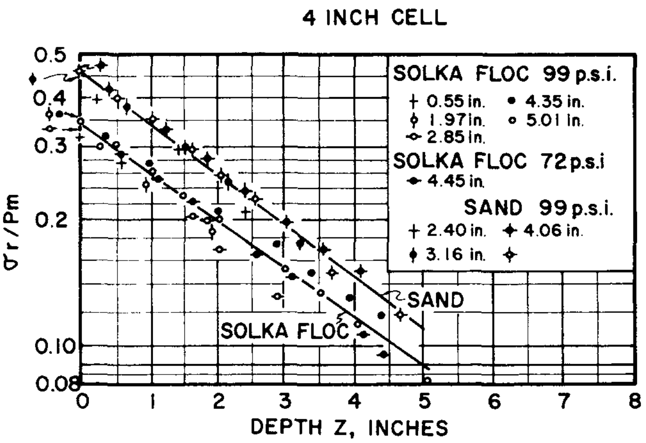

Fig. 3. Logarithm of sidewall pressure ratio versus depth.

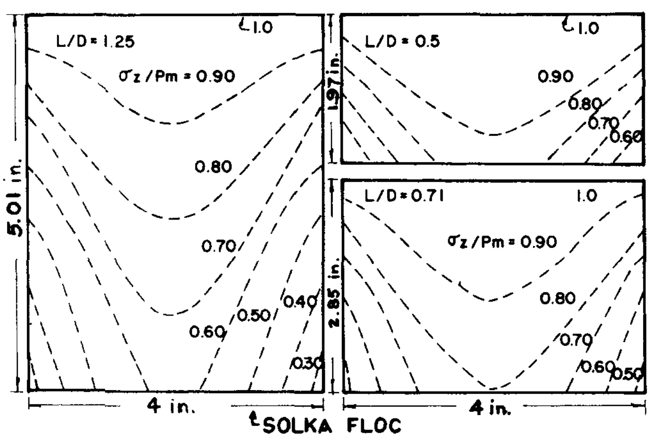

Fig. 4. Qualitative estimate of vertical pressure variation in C-P cell.

Assuming K_0 to be constant, the vertical pressures at the wall can be determined by dividing the extrapolated values of σ_r by K_0. With measured values of σ_z on the septum, all of the values of σ_z at the boundaries of the cake are known. Qualitatively it becomes possible to estimate lines of constant σ_z inside the cake.

In Figure 4 a qualitative view of the variation of vertical pressure is shown for a number of cakes in 4-in. cells. It is perfectly clear that the cakes are quite nonuniform and that σ_z is not constant at a given z as assumed in the simplified theory presented in Part VII (*18*) of this series. Nonuniformity in the commonly used 2-in. cells would be even worse.

If porosity and permeability are unique functions of σ_z, the lines for constant σ_z will also represent constant porosity and permeability. Although it has been assumed that porosity and permeability are determined by the solids compressive pressure alone in conventional theory, there is no information to indicate how they will vary with changing conditions of shear and horizontal pressure. For example, in a triaxial test as used in soil mechanics, σ_z and σ_r can be varied independently. It is probable that both porosity and permeability would be affected by such independent variations.

Assuming that σ_z, σ_r, porosity, and permeability vary throughout the cake, the stream lines for flow of the fluid will not be parallel and straight. While no attempt was made to measure radial variations of the liquid leaving the septum, the results of Figure 4 give strong evidence for nonuniform fluid flow.

Similar curves were obtained for the sand indicating that the stress distributions were similar in both rigid and highly compressible media. However, with the sand, the porosity and structure of the cake would not be affected by moderate stress nonuniformity, provided that the particles were not deformed. Consequently, porosity and permeability measurements would provide reliable results for incompressible materials. Both porosity and permeability would be independent of applied pressure in the latter case conclusions.

The data presented in this article point to the necessity for a more thorough analysis of the use of C-P cells for predicting point values of resistance and porosity. Information is needed concerning different solids and different coating materials for cell walls. Some materials will give more uniformity than others.

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NOTATION

D = diameter of the cell, ft
K_0 = ratio of horizontal to vertical stress
L = cake thickness, ft
P_m = applied pressure, lb_f/ft^2
r = radial distance from center, ft

R = radius of cell, ft
z = distance from the cake surface, ft
σ_r = horizontal component of stress, lb_f/ft^2
σ_z = vertical component of stress, lb_f/ft^2

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The Dynamics of a Packed Gas Absorber by the Pulse Response Technique

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The dynamic response of a packed gas absorber to inlet gas composition changes was investigated by the pulse response technique for the case where absorption was accompanied by pseudo first-order irreversible chemical reaction.

The solute-carrier-solvent system was carbon dioxide-air-0.07 normal aqueous sodium hydroxide solution. The column used was 6.5 in. I.D. Plexiglass packed to a height of 6.15 ft. with either ¼-in. Raschig rings or ¾-in. Berl Saddles. Gas flow rates ranged from 1.5 to 10 lb.-moles/hr./sq.ft. and liquid flow rates ranged from 0 to 200 lb.-moles/hr./sq.ft. A pulse of carbon dioxide was injected into the inlet air stream and monitored as it entered the packed section. The outlet gas and liquid phase concentrations were continuously monitored as the resulting pulses left the packed section. The pulse response data were reduced to frequency response by digital computer calculations. Reliable data could be obtained over the frequency range 0 to 5 radians/second.

The experimental results were compared with theoretical predictions from the slug flow, axial diffusion, and mixing cell models. Both the mixing cell model and the axial diffusion model satisfactorily predicted the experimental frequency responses over the entire frequency range covered. The slug flow model was found unsatisfactory for predicting gas-phase amplitude ratios at high frequencies, where axial mixing affected the amplitude ratios. Gas-phase particle Peclet numbers and overall mass transfer coefficients based on the gas-phase driving force determined in the present absorption system were in reasonable agreement with the values reported in the literature.

The rational design of process control systems, the development through simulation of effective plant start-up procedures, and the proper sizing of equipment for unsteady state applications require valid mathematical models describing the dynamic behavior of each unit in the process system. Over the past fifteen years a number of papers